Pandemics

Problem solved by the students of “Liceo Nievo” in Padua

1.1 Variables and parameters

We set with N as the total population, formed by the number of infected people (M_t), number of healthy people (S_t), number of immune people (I_t), number of dead people (D_t) in an instant t (days). All people who get out of their illness become immune to it.

We also consider the following parameters:

- Healthy
  - α is the possibility of getting sick
  - η is the factor of natural birth
  - μ is the factor of natural death
- Infected
  - β is the probability of death due to illness
  - γ is the factor of recovery
- Immune
  - δ is the factor of immune natural birth

1.2 Equations

The equations that describe the progress of population in time are:

$$\begin{cases} S_{t+1} = S_t + \eta S_t - \mu S_t - \alpha S_t M_t \\ M_{t+1} = M_t + \alpha S_t M_t - \beta M_t - \gamma M_t - \mu M_t \\ I_{t+1} = I_t - \mu I_t + \delta I_t + \gamma M_t \end{cases}$$

To simplify the problem, natural births and deaths may be overlooked, placing $\delta = \mu = \eta = 0$.

This way we obtain a simplified system:

$$\begin{cases} S_{t+1} = S_t - \alpha S_t M_t \\ M_{t+1} = M_t + \alpha S_t M_t - \beta M_t - \gamma M_t \\ I_{t+1} = I_t + \gamma M_t \end{cases}$$

1.3 Probability of contraction

Assuming that in average one people only meets another one once a day, *N* people will produce $\binom{N}{2} = \frac{N(N-1)}{2}$ encounters per day.

One of these contacts is to be considered at risk only between infected and healthy people, producing M_tS_t risky encounters. The probability of having a risky encounter is the division between risk encounters and total encounters, producing the formula $\frac{2S_tM_t}{N(N-1)}$.

Since the possibility of getting infected is α, experimental constant, every day $\alpha \cdot \frac{2S_tM_t}{N(N-1)}$ contagions happen, obtaining the following formula:

$$S_{t+1} = S_t - \frac{2\alpha S_t M_t}{N(N-1)}$$

1.4 Recovery probability

The first day the recovery probability is *γ*, the second day it increases to *(1-γ)γ*; the third day *(1-γ)(1-γ)γ*. Generally speaking, in the day *k* the probability is $\gamma(1-\gamma)^{k-1}$.

If the first day the value was *γ = 0.2*, the second day it would be *γ = 0.2 + (1-0.2)0.2 = 0.36*. This is due to the fact that *1-γ* represents everyone that did not recover in day 1 timed for *γ* unchanged in all days.

An intuitive method to demonstrate that probability and time *T* are intertwined consists of analysing the sum of every recovered *γM*. If *T = 4,* then *γM + γM + γM + γM = M* and so $\gamma = \frac{1}{4}$ and *γ* would be $\gamma = \frac{1}{T}$.

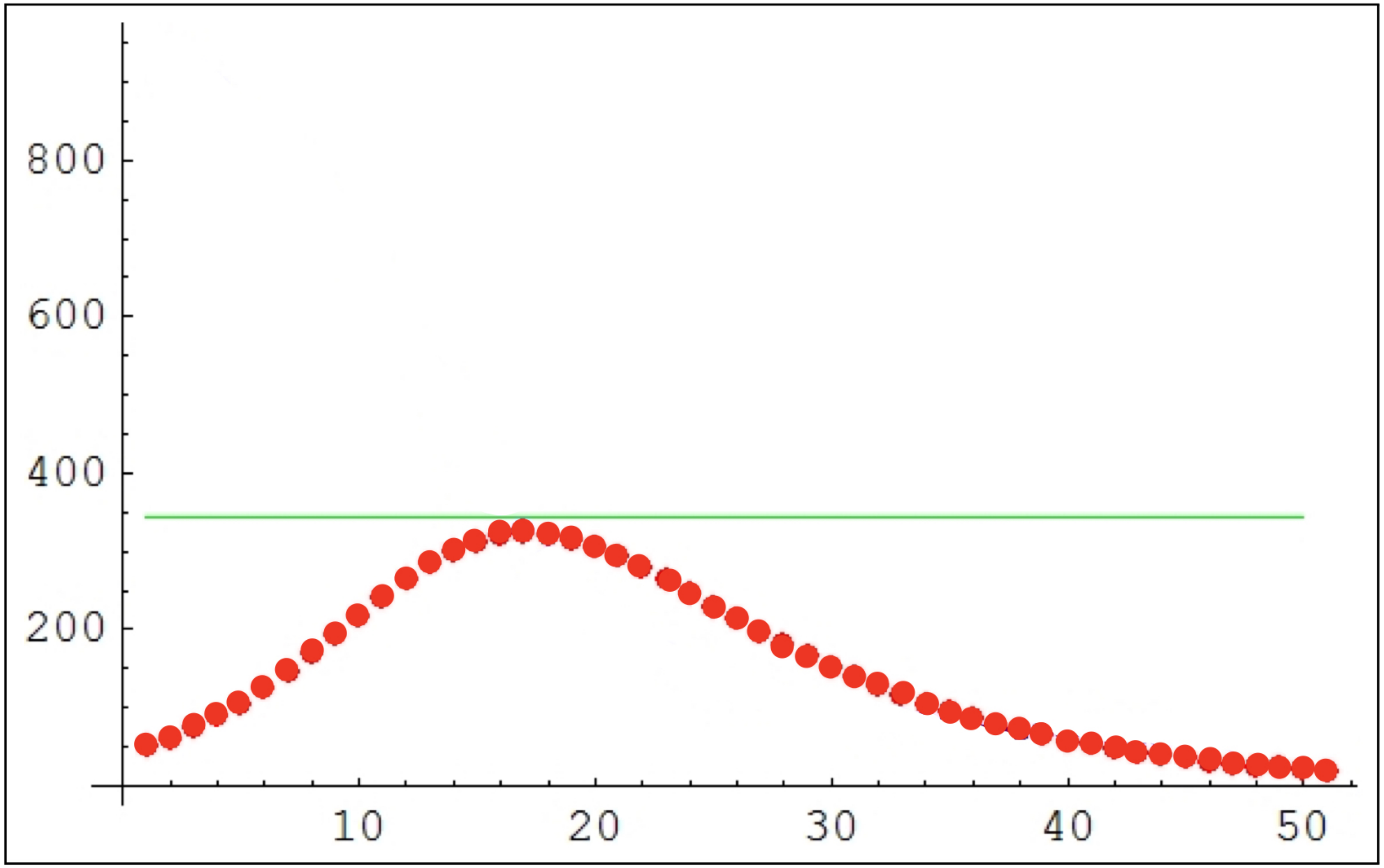
1.5 Function trend

If the division/rapport between recovery and lethality is higher than 1, the number of illness will be inclined towards 0; if it lower than 1, the number of immune people would tend to 0.

In the first case, in chart represented with time and number of sickened/ infected people, the function would initially increase, then decreasing to 0.

In the second case the same exact thing happens with the immunes but changes considering how healthy people increase or decrease in the population.

If the division is 1 the function will fluctuate irregularly about an horizontal asymptote.



First case: the function would initially increase, then decreasing to 0.